2 Powers

1 Power

It is a number obtained by multiplying a number by itself a certain number of times

For example $3^4 = 3x3x3x3 = 81$

How to name powers.

6^5 Is read as

- **Six to the fifth power**
- **Six to the power of five**
- **Six powered to five.**

The most common is **six to the power of five**

6 is the **base**

5 is the **index** or **exponent**

Especial cases: **Squares and cubes** (powers of two and three)

3^2 is read as:

- **Three to the second power**
- **Three squared**
- **Three to the power of two.**
- **Three to the square power**

The most common is **Three squared**

5^3 Is read as:

- **Five cubed**
- **Five to the third power**
- **Five to the power of three.**

The most common is **Five cubed**

Exercises

1. Calculate mentally and write in words the following powers:

a) 4^3 = ______________________

b) 5^4 = ______________________

c) 11^2 = ______________________

d) 2^5 = ______________________

e) 5^3 = ______________________

f) 100^2 = ______________________

g) 10^3 = ______________________

2. Match the following numbers to their squares.

169 196 25 81 400 10000

20^2 13^2 9^2 5^2 100^2 14^2

2.1 Write the square of the fifteen first numbers.

2.2 Write the ten first cubes.

3. Describe the pattern formed by the last digit of any square number.

- **Are there any numbers that do not appear as the last digits?**

- **Could** 413 **be a square number?**

4. Match the following numbers to their cubes.

125 8000 1 64 1000 27

20^3 4^3 10^3 3^3 5^3 1^3

5. Find and read the whole expression:

a) $3^3 =$ **b)** $6^3 =$ **c)** $600^2 =$ **d)** $2^7 =$ **e)** $1000^3 =$

6. Fill in the missing numbers.

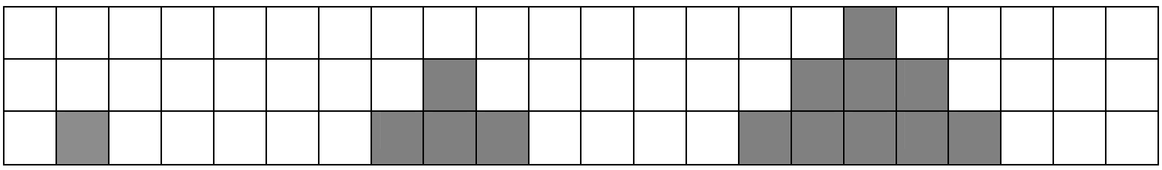
a) $5x5x5x5x\ 5 = 5^{[\ ]}$ **b)** $8x8x8x8 = 8^{[\ ]}$ **c)**$10000000 = 10^{[\ ]}$

d) $81 = [\ ]^4$ **e)** $16 = [\ ]^2$ **f)** $16 = 2^{[\ ]}$

7. Write a list with the squares of all the whole numbers from 1 to 12

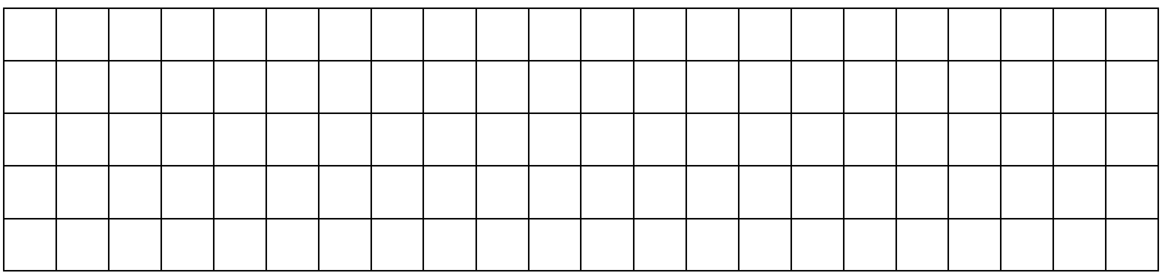
8. Write a list with the cubes of all the whole numbers from 1 to 10

9. You can build up a pattern using square tiles.



Shape 1 **Shape 2** **Shape 3**

a) Draw the next two shapes in the pattern.



b) Count the numbers of tiles in each shape and put your results in a table

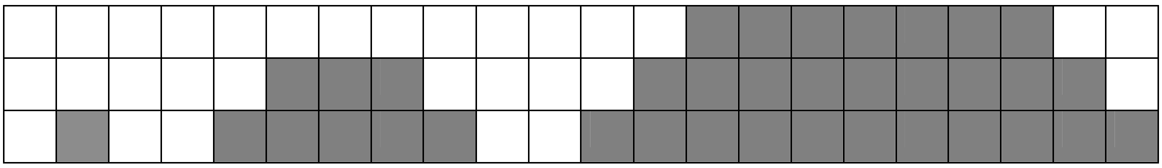
Shape number	**1**	**2**	**3**	**4**	**5**	
Tiles						

c) How many tiles would be in:

1. Shape 6 **2. Shape 9** **3. Shape 15**

d) Without drawing it, explain how to know the number of tiles when you know the number of the shape.

10. This pattern is built up using square tiles.



1 3 + 5 7+9+11

Shape 1 **Shape 2** **Shape 3**

e) Draw the next two shapes in the pattern.

f) Count the numbers of tiles in each shape and put your results in a table

Shape number	**1**	**2**	**3**	**4**	**5**	
Tiles						

g) How many tiles would be in:

1. Shape 5 **2. Shape 7** **3. Shape 10**

h) Without drawing it, explain how to know the number of tiles when you know the number of the shape.

11. Some numbers are equal to the sum of two squares, for example

$$3^2 + 5^2 = 34.$$

Which numbers smaller than 100 are equal to the sum of two squares?

How many of them are equal to the sum of two squares in more than one way?

Big numbers can be written using powers of 10, for example $70000 = 7x10^4$, $123000000 = 1.23x10^8$ this form of writing numbers is called **standard form.**

Exercise 12. Express in standard form the following numbers:

a) **4,000,000,000**

b) **A billion**

c) **321,650,000 (round to the million)**

d) **The length of the earth meridian in metres (round appropriately)**

e) **The number of seconds in a year (round appropriately)**

Exercise 13. Write as ordinary numbers

a) $3.4x10^5$ **b)** $0.05x10^2$ **c)** $2.473x10^8$

d) $7.26x10^2$ **e)** $7.006x10^7$ **f)** $9x10^{12}$

2 Operations with powers

To manipulate expressions with powers we use some rules that are called laws of powers or **laws of indices**.

2.1 Multiplication:

When powers **with the same base** are multiplied, the base remains unchanged and the **exponents are added**

Example:

$$7^5x7^3 = (7x7x7x7x7)x(7x7x7) = 7^8$$

So $7^5x7^3 = 7^8$

Exercise 14. Fill in the missing numbers.

a) $3^3 \cdot 3^4 = (3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3) = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^{[\ ]}$

b) $7^5 \cdot 7^8 = 7^{[\quad]}$ **c)** $6^2 \cdot 6^7 = 6^{[\ ]}$ **d)** $6^5 \cdot 6^{[\quad]} = 6^9$

e) $[\ ]^3 \cdot [\ ]^4 = 2^7$ **f)** $2^5 \cdot 2^{[\ ]} = 2^6$ **g)** $2^7 \cdot [\ \ ]^{[\ ]} = 2^9$

2.2 Division:

When powers **with the same base** are divided, the base remains unchanged and the **exponents are subtracted.**

Example:

$$6^7 : 6^4 = \frac{6x6x6x\not{6}x\not{6}x\not{6}x\not{6}}{\not{6}x\not{6}x\not{6}x\not{6}} = 6x6x6 = 6^3$$

So $6^7 : 6^4 = 6^3$

Exercise 15. Fill in the missing numbers.

a) $7^5 : 7^2 = 7^{[\ ]}$ **b)** $12^{13} : 12^{\ 7} = 12^{[\ ]}$ **c)** $3^8 : [\ \ ]^3 = 3^5$

d) $5^{[\ ]} : [\ \ ]^2 = 5^7$ **e)** $3^8 : [\ \ ]^3 = 3^5$ **f)** $[\ \ ]^{12} : [\ \ ]^9 = 9^{[\ ]}$

2.3 Power of a power:

The exponents or indices must be multiplied

Example:

$$\left(2^3\right)^5 = \left(2^3\right)x\left(2^3\right)x\left(2^3\right)x\left(2^3\right)x\left(2^3\right) = 2^{3+3+3+3+3} = 2^{3x5} = 2^{15}$$

So $\left(2^3\right)^5 = 2^{15}$

Exercise 16. Fill in the missing numbers.

a) $\left(4^2\right)^5 = 4^{[\ ]}$ **b)** $\left(3^2\right)^{[\ ]} = 3^8$ **c)** $\left(3^{[\ ]}\right)^2 = 3^8$

d) $\left([\ \ ]^2\right)^3 = 5^6$ **e)** $\left(2^2\right)^{[\ ]} = [\ \ ]^8$

2.4 Powers with different base but the same exponent

Multiplication: When powers **with the same exponent** are multiplied, multiply the bases and keep the **same exponent.**

Example:

$$2^5 x 7^5 = (2x7)^5 = 14^5$$

Division: When powers **with the same exponent** are divided, bases are divided and the **exponent remains unchanged.**

Example:

$8^3 : 2^3 = (8:2)^3 = 4^3$, we can also use fractions notation $\frac{8^3}{2^3} = \left(\frac{8}{2}\right)^3 = 4^3$

Exercise 17. Fill in the missing numbers.

a) $3^7 \cdot 8^7 = [\quad]^7$ **b)** $[\quad]^2 \cdot [\quad]^2 = 6^2$ **c)** $5^2 \cdot [\quad]^2 = 15^2$

d) $2^2 \cdot [\quad]^2 = 14^{[\quad]}$ **e)** $8^5 : 4^5 = [\quad]^5$ **f)** $\frac{15^5}{5^5} = [\quad]^5$

g) $\frac{16^7}{8^7} = [\quad]^7$ **h)** $\frac{6^{12}}{3^{12}} = [\quad]^{[\quad]}$

Exercise 18. Operate.

a) $7^3 \cdot 7^5 = 7^{[\quad]}$ **b)** $4^2 \cdot 4^3 \cdot 4^6 \cdot 4 =$ **c)** $9^2 \cdot 9^7 \cdot 9^2 =$

d) $5^7 : 5^3 =$ **e)** $(2^2 \cdot 2^6) : 2^3 =$ **f)** $(14^2)^4 =$

g) $3^7 : (3^2 \cdot 3^3) =$ **h)** $(2^2 \cdot 3^2) = [\quad]^2$ **i)** $(12^2 \cdot 12^3)^4 =$

j) $(6^2 : 3^2) = [\quad]^2$ **k)** $[3^8 : (3^2 \cdot 3^3)]^2 = 3^{[\quad]}$ **l)** $\frac{5^7 \cdot 5^3}{5^4} =$

3 Square roots.

The inverse operation of power is root.

The inverse of a square is a square root, that is: If we say that $\sqrt{9} = 3$, that means that $3^2 = 9$

10. Calculate the following square roots:

a) $\sqrt{81} =$ **b)** $\sqrt{121} =$ **c)** $\sqrt{100} =$ **d)** $\sqrt{10000} =$

e) $\sqrt{900} =$ **f)** $\sqrt{1600} =$ **g)** $\sqrt{250000} =$

There are numbers that are not squares of any other number, for example between 9 and 16 (squares of 3 and 4); there is not any whole square number.

The square root of all numbers between 9 and 16 are between 3 and 4, for example

$3 < \sqrt{13} < 4$, this is an estimation of the $\sqrt{13}$ value.

Exercise 19. Estimate the value of the following square roots:

a) $< \sqrt{57} <$ **b)** $< \sqrt{250} <$ **c)** $< \sqrt{700} <$

d) $< \sqrt{1500} <$ **e)** $< \sqrt{30} <$ **d)** $< \sqrt{2057} <$

When we estimate a square root as $3 < \sqrt{13} < 4$ we can also say that $\sqrt{13}$ is 3 and the difference of 13 and 9 (square of 3), which is 4, is called the **remainder.**

So we say that the **square root of 13 is 9 and the remainder is 4.**

That means $13 = 3^2 + 4$

Exercise 20. Calculate the square roots and the remainders for the numbers of the previous exercise; write the meanings as in the example.